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PREDICTING PHASE NOISE IN CRYSTAL OSCILLATORS

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Abstract - **An enhanced phase noise model [1] allows to predict the phase noise in crystal oscillators. It is based on an improvement of the mathematical analysis proposed by G. Sauvage [2]. In this model, power spectral densities of phase fluctuations are computed in different points of the oscillator loop. They are calculated from their correlation functions. The resonator caused noise as well as the amplifier caused noise are taken into account and distinguished.**

In order to validate this model, the behavior of about ten of 10 MHz quartz crystal oscillators is observed and analyzed. These oscillators have been chosen in a facility production. Resonators have been selected according to the value of their resonant frequency and their motional resistance. We attempt to measure separately the amplifier and resonator caused noises by means of a passive method. The phase noise of the overall oscillator working in closed loop is provided by the usual active method. Theoretical and experimental results are compared and discussed.

Keywords – Phase noise, quartz crystal oscillator

I. INTRODUCTION

The focus of this paper is to validate a simple and efficient tool for predicting the noise of an oscillator from the amplifier and resonator caused noises. The analysed model is derived from [1]. Experiments are performed with various batches of medium cost OCXOs (resonators and electronics) extracted from common industrial production. Resonators are 10 MHz SC-cut in HC-43 enclosures. They are ovenized with their associated oscillator printed circuit board into 25 mm long, 25 mm large and 12 mm high packages.

Why a new model? In fact this model is an enhanced version of the Leeson's model which has the following limitations [3]. First, in the Leeson's model the amplifier gain is assumed to remain a constant versus frequency close to the carrier frequency. Second, in the Leeson's model the filter transfer function is considered to be a symmetrical one on each side of the carrier frequency. This is available for quartz crystal oscillator whose resonator is working on its resonant frequency. In that case, the resonator looks like a pure resistor at the oscillator frequency. Actually, this is not the case in a one-transistor oscillator where the resonator usually works as an inductor in the pass band filter.

A quick review of this new model is given before testing it from experiments.

II. PREDICTING MODEL OF THE PHASE NOISE IN CRYSTAL OSCILLATORS

Basically, the oscillating loop is divided in its two usual components: the amplifier, whose transfer function is here named $G(\nu)$, and the pass band filter including the quartz crystal resonator, which has a transfer function denoted $H(\nu)$. In the following, $g(t)$ and $h(t)$ would be their corresponding functions in the time domain.

A. Open Loop phase noise

Fig. 1 shows an open loop version of the oscillator, available for phase noise calculation. One can observe that functions which are used in Fig. 1 are not exactly $g(t)$ and $h(t)$ but the "normalized" impulse responses of the amplifier and of the filter, $\overline{g}(t)$ and $\overline{h}(t)$ respectively.

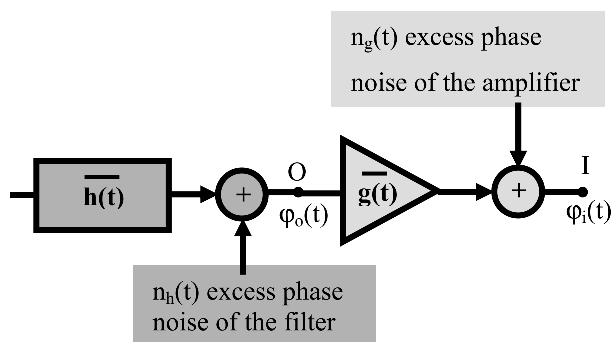


Fig. 1. Basic open loop circuit.

Their Fourier transforms are:

$$\overline{G}(\nu) = \frac{G(\nu)}{G(\nu_c)} \text{ and } \overline{H}(\nu) = \frac{H(\nu)}{H(\nu_c)}$$

where ν_c is the carrier frequency here equal to the oscillator frequency ($G(\nu_c)\cdot H(\nu_c) = 1$ in steady state). Indeed, phase noise is relative to the carrier power. Thus, it is convenient to define this normalized set of functions[1].

Then, from Fig. 1, it is easy to show that the Power Spectral Density (PSD) $S_{\varphi i}^{OL}$ of phase fluctuations $\varphi_i(t)$ at point I is:

$$S_{\varphi i}^{OL}(\nu) = \left|\overline{G}(\nu)\right|^2 \cdot S_{nh}(\nu) + S_{ng}(\nu) \quad (1)$$

where $S_{nh}(\nu)$ and $S_{ng}(\nu)$ are the filter caused noise and amplifier caused noise PSDs respectively.

B. Closed Loop phase noise

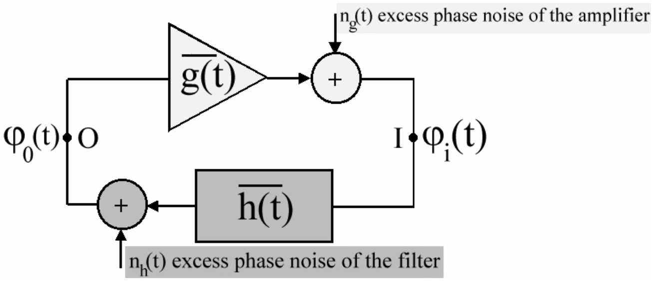


Fig. 2. Resulting closed loop circuit.

When closing the open loop shown in Fig. 1 it looks like Fig. 2. For this Closed Loop, the PSD of phase fluctuations $S_{\varphi i}^{CL}$ of $\varphi_i(t)$ can be expressed in terms of PSDs of both noises $n_h(t)$ and $n_g(t)$ [1]:

$$S_{\varphi i}^{CL}(\nu) = \frac{1}{\left|\overline{G}\cdot\overline{H}(\nu)-1\right|^2}\cdot\left[\ \left|\overline{G}(\nu)\right|^2\cdot S_{nh}(\nu) + S_{ng}(\nu)\ \right] \qquad (2)$$

That is to say:

$$S_{\varphi i}^{CL}(\nu) = \frac{1}{\left|\overline{G}\cdot\overline{H}(\nu)-1\right|^2}\cdot S_{\varphi i}^{OL}(\nu) \qquad (3)$$

These PSDs are usually expressed in dBc/Hz

III. Measurement of the resonator phase noise

Phase noise of a couple of quartz crystal resonators (QX) is measured in a commercial version of carrier suppression bench [4] (Fig. 3).

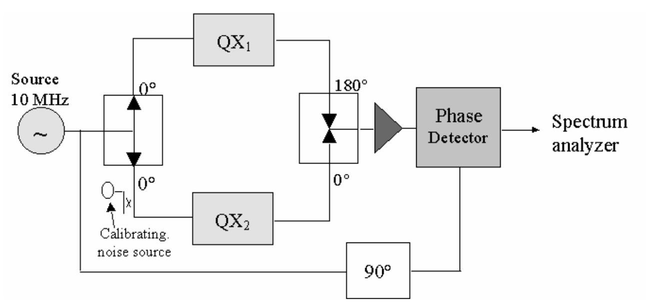


Fig. 3. Resonator measurement system.

Each arm is tuned at the source frequency. Typically, crystal frequencies are 50 Hz or 20 Hz below 10 MHz depending on the resonator batches. Dissipating powers into QXs are also adjusted according to their working powers in oscillator. Moreover, each QX is separately temperature-controlled in order to work at its own turn over temperature. Fig. 4 shows various phase noise records of SC-cut resonator samples selected into manufactured batches. These resonators exhibit typical quality factors of about $Q \approx 1.2\cdot10^6$ and motional resistors $R \approx 85\ \Omega$. Three kinds of finishing processes of crystal surfaces have been tested. As shown in Fig. 4, a lower phase noise level is achieved when the resonator surfaces are delicately prepared. Although this result seems to be obvious, it is worth verifying if one step of the process can or cannot be avoid.

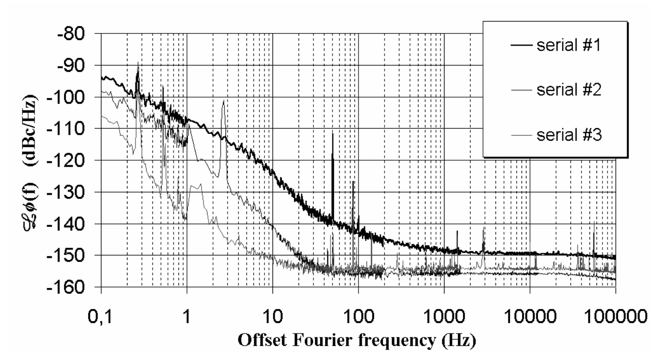


Fig. 4. 10 MHz resonator measurements.

The carrier suppression bench shown in Fig. 3 could be used to measure phase noise of active elements such as oscillators amplifier. This would require modifications of our standard system in terms of electronics because of the load matching and essentially on the mechanical aspect because of the ovens. These modifications have not still been done. So, as discussed below, measurements of the open loop noise were limited by the use of a less sensitive system.

IV. Comparison between loop measurements and analytical modeling

A. Oscillator description

The analyzed oscillator can be classified as a Clapp oscillator and simply described with Fig. 5.

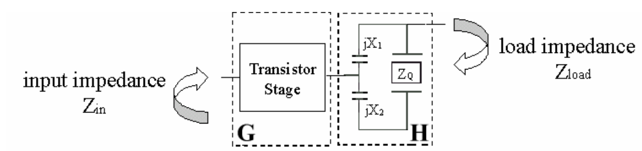


Fig. 5. Oscillator description.

Analytically, the main simplified relationships have the form:

$$G \approx h_{fe}\cdot(RE\ //\ Z_{load})/(h_{ie} + h_{fe}\cdot(RE\ //\ Z_{load}) \qquad (4\text{-a})$$

$$H \approx Z/(Z + jX_1)\ \text{with}\ Z = Z_Q\ //\ Z_{in} \qquad (4\text{-b})$$

where // means in parallel, h_{ie} and h_{fe} are the small signal input resistor and current gain of the transistor stage, RE is for the emitter resistor, $Z_{in} = Z_{load}$ the input impedance of the open loop circuit, or its load impedance depending on the point of view. All these quantities can be computed from SPICE simulations. The most important point for a SPICE analysis as well as an analytical one is to take into account adequately the impedance loading $Z_{load} = Z_{in}$ due to the loop closing. Obviously, the oscillating frequency ν_c is that for which the phase of the loaded function $G{\cdot}H$ is zero (modulo 2π) provided that its modulus is larger than 1.

In term of noise, the other way to open the loop is shown in Fig. 6.

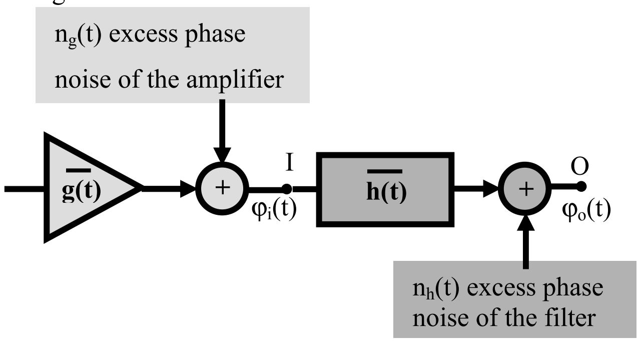


Fig. 6. Open loop circuit.

In this case the open loop PSD of phase fluctuations at point O is:

$$S_{\varphi o}^{OL}(\nu) = S_{nh}(\nu) + \left|\overline{H}(\nu)\right|^2 \cdot S_{ng}(\nu) \qquad (5)$$

The resulting PSD at this point is still given by:

$$S_{\varphi o}^{OL}(\nu) \times \frac{1}{\left|\overline{G} \cdot \overline{H}(\nu) - 1\right|^2} = S_{\varphi o}^{CL}(\nu) \qquad (6)$$

Let us defined $f = |\nu - \nu_c|$ the offset Fourier frequency from the carrier frequency. Then, the predicted oscillator noise $S_{\varphi o}^{CL}(\nu)$ expressed in dBc/Hz is equal to the open loop noise including both the amplifier and the resonator caused noise plus $10 \cdot \mathrm{Log}_{10}\left(\frac{1}{\left|\overline{G} \cdot \overline{H}(\nu) - 1\right|^2}\right)$, as illustrated in Fig. 7.

This last term should be divided in one part for the positive side band plus another part for the negative side band (see Fig. 7b.). Usually the symmetry is assumed that is to say an increase of 3dB from the first part.

As shown in Fig. 7a, the loop effect consists of an excess noise of more than 10 dB on each side of the carrier, in our case, plus the well-known increase of 20 dB per decade close to the carrier in the filter bandwidth.

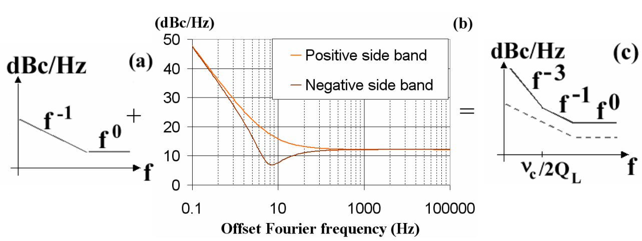


Fig. 7. Noise transform through the loop, (a) open loop DSP in dBc/Hz, (b) $\frac{1}{\left|\overline{G} \cdot \overline{H}(\nu) - 1\right|^2}$ function, (c) Resulting closed loop DSP.

B. Measurements

For the reasons mentioned above the carrier suppression bench (see Fig. 3) has not been modified in order to measure noise of active devices. It has been used for resonator noise measurements only.

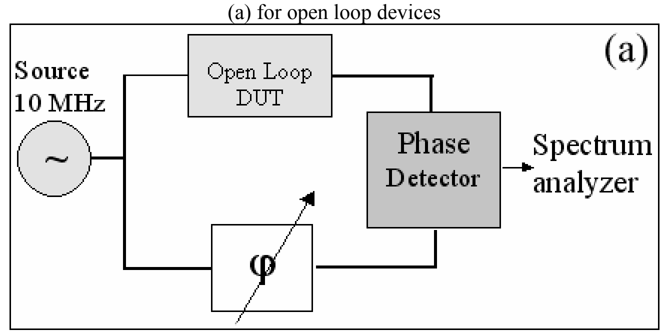
We attempt to measure phase noise of loop amplifiers using the conventional set-up shown in Fig. 8a. This was unsuccessful because of the insufficient sensitivity of the system. Indeed, loading the amplifier by 50 Ω leads to a low level at the phase detector input. As a consequence its sensitivity decreases and the noise floor of the system increases. As an example an input level of –20 dBm leads to a mixer sensitivity of 0.02 V/rd which increases the noise floor of +15 dB. Amplifiers tested separately or with their associated resonators exhibit too low level voltages when measured in open loop in the operating conditions of the oscillator closed loop. Then, the bench cannot detect noise PSDs lower than –155 dBc/Hz.

In other words, phase noise of amplifiers loaded with 50 Ω is commonly lower than the noise floor of our measuring system. An impedance matching would be necessary provided that it does not introduce extra phase noise. Actually, cross correlation methods should be used [5][6].

Nevertheless, although noise measurements of separated amplifiers are not possible, noise measurements of the entire open loop device have been successfully performed with the bench of Fig. 8a.

When closing their loop these devices become oscillators which can be analyzed in terms of phase noise using the other conventional set-up shown in Fig. 8b.

(a) for open loop devices



(b) for closed loop devices

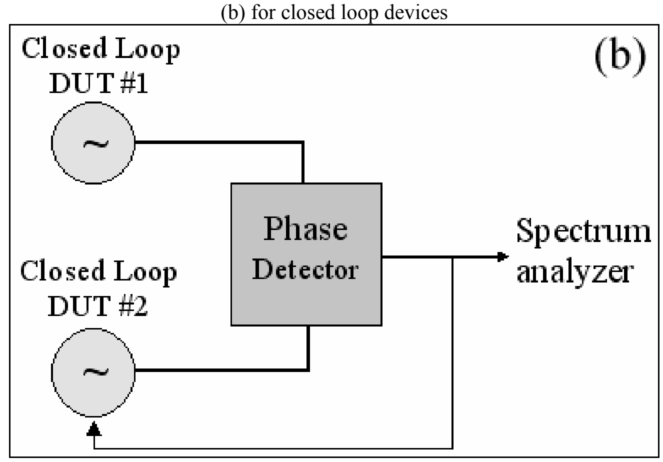
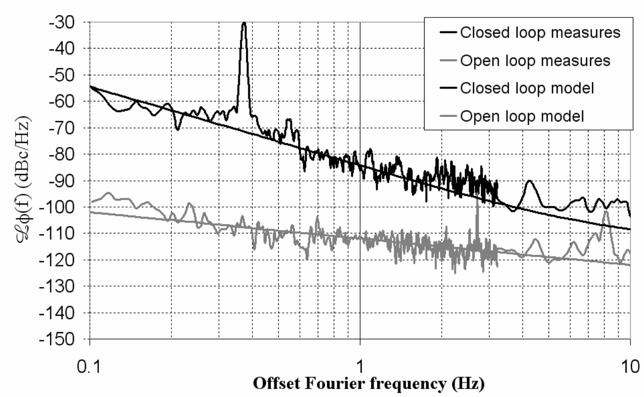


Fig. 8. Bench used for the phase noise measurement
(a) for open loop devices, (b) for closed loop devices.

Major results are summed up in Fig. 9. Experimental data agree with theory:

(a) f^{-1} to f^{-3} transform



(b) White noise increasing

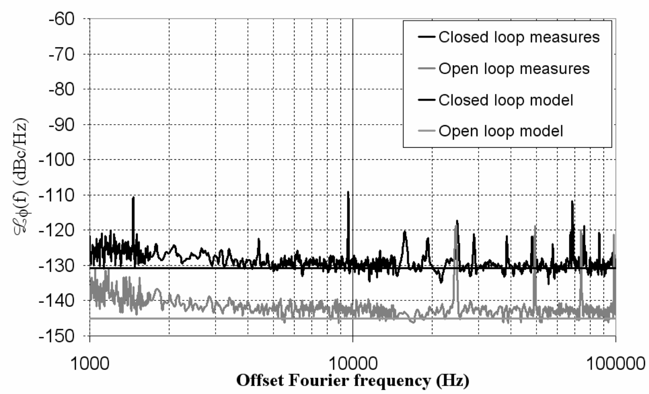


Fig. 9. Comparison between experimental and theoretical data (a) f^{-1} to f^{-3} transform, (b) White noise increasing.

- As expected, f^{-1} noise of the open loop is transformed through the loop to f^{-3} noise within the loaded quartz crystal bandwidth and is up-translated (see also Fig. 7a.).

- Moreover, noise is globally increased (Fig. 5a. and b.) by the $\dfrac{1}{\left|\overline{G}\cdot\overline{H}(\nu)-1\right|^2}$ factor that was not taken into account in the Leeson's model.

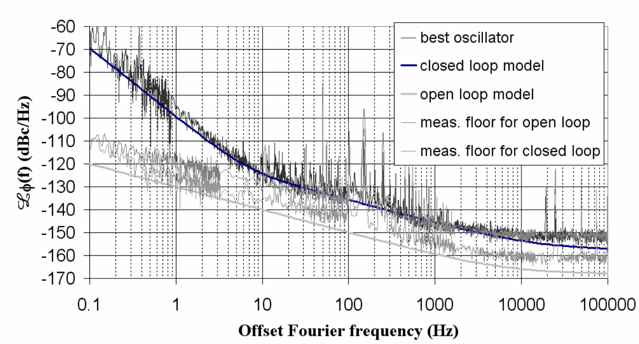


Fig. 10. Best tested oscillator, noise floor limits and theoretical results.

Measurements were deliberately performed on medium quality resonators and oscillators in order to eliminate the limitation of the experimental noise floor mentioned above. Moreover this makes easier the comparison with our theoretical model. In our operating conditions, oscillators including the best manufactured resonators cannot be tested in open loop, as shown in Fig. 10 where various noise floors are specified.

V. Conclusion

This work was the opportunity to validate a theoretical model from manufacturing batches of medium cost resonators and oscillators. This is a good way to reach reproducibility of results.

Unfortunately, performances of the measurement method are limited to medium quality resonators in the case of open loop circuits. Because of 50 Ω loads, amplifier outputs exhibit very low signal levels which reduce the phase detector sensitivity of the bench.

Nevertheless, when measurements are possible, good agreement has been observed between predicted and practical results. The predicted noise offset of the closed loop PSD of phase fluctuations compared with the open loop one is emphasized in this work. The Leeson's model cannot show this phenomenon which is particularly visible on the noise floor.

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